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EVOLUTION OF THE DIFFUSION MIXING LAYER OF TWO GASES UPON INTERACTION WITH SHOCK WAVES

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A mathematical model of mechanics of a two-velocity two-temperature mixture of gases is developed. Based on this model, evolution of the mixing layer of two gases with different densities under the action of shock and compression waves is considered by methods of mathematical simulation in the one-dimensional unsteady approximation. In the asymptotic approximation of the full model, a solution of an initial-boundary problem is obtained, which describes the formation of a diffusion layer between two gases. Problems of interaction of shock and compression waves with the diffusion layer are solved numerically in the full formulation. It is shown that the layer is compressed as the shock wave traverses it; the magnitude of compression depends on shock-wave intensity. As the shock wave passes from the heavy gas to the light gas, the mixing layer becomes overcompressed and expands after shock-wave transition. The wave pattern of the flow is described in detail. The calculated evolution of the mixing-layer width is in good agreement with experimental data.

Key words: *shock wave, mixing layer, Richtmyer–Meshkov instability, two-velocity two-temperature gas-dynamics of mixtures.*

Introduction. The mixing layer is traditionally considered as a density-discontinuity surface, i.e., as a contact discontinuity. Interaction of a shock wave with a contact discontinuity in the one-dimensional unsteady approximation is described by the classical solution of the problem of decay of an arbitrary discontinuity. Shock-wave transition from one gas to the other through a disturbed contact discontinuity generates the Richtmyer–Meshkov instability. A region of turbulent mixing separating compressed gas flows is formed at the final stage in the region of the original contact discontinuity. It is known that replacement of a stepwise velocity profile on the contact discontinuity by a continuous distribution in a certain finite-width layer can reduce the growth rate of disturbances at the initial stage of development of the Richtmyer–Meshkov instability. This was noted, e.g., in [1, 2], where the growth rate of disturbance amplitude was considered theoretically, and in experimental works [3–6].

The theoretical studies on the basis of gas-dynamic equations were mainly qualitative, and the mixing layer was modeled by a layer with a variable density. Therefore, it is of interest to consider this problem on the basis of the equations of a two-velocity two-temperature mixture of gases, where each component has its own velocity and temperature. This approach allows one to describe both the processes of interpenetration of gases and mixing layer–shock wave interaction. The necessity of using models of multispecies mixtures for the description of decay of the contact boundary and formation of the mixed layer was noted in [7]. A semi-empirical model of turbulent mixing of a multispecies medium was developed in [8]; the model is based on the use of a specific velocity for each component and implies that turbulent mixing begins immediately. Processes at the initial stages of mixing are considered below with the help of equations of two-velocity two-temperature gas dynamics.

Mathematical Model. We study the evolution of the transitional layer separating two pure gases with different densities within the model of one-dimensional unsteady flow of the mixture with a shock wave acting on the layer. The parameters of the mixture in the layer are described by equations of two-velocity two-temperature gas dynamics of mixtures [9]:

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$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i u_i}{\partial x}=0,\qquad \rho_i\frac{\partial u_i}{\partial t}+\rho_i u_i\frac{\partial u_i}{\partial x}+\frac{\partial p_i}{\partial x}=K(u_j-u_i),$$

$$\rho_i\frac{\partial e_i}{\partial t}+\rho_i u_i\frac{\partial e_i}{\partial x}+p_i\frac{\partial u_i}{\partial x}=K\beta_{ij}(u_j-u_i)^2+q(T_j-T_i), \tag{1}$$

$$p_i=kn_iT_i,\qquad e_i=c_{iv}T_i,\qquad \rho_i=m_in_i,\qquad i,j=1,2,\quad i\neq j.$$

Here ρ_i, u_i, e_i, p_i, T_i, and m_i are the density, velocity, internal energy, pressure, temperature, and weight of a molecule of the ith kind, k is the Boltzmann constant, $K=16\rho_1\rho_2\Omega_{12}^{(1,1)}/(3(m_1+m_2))$, $\Omega_{12}^{(1,1)}$ is the collision integral, $\beta_{ij}=m_iT_i/(m_iT_i+m_jT_j)$, $q=3m_1K/(m_1+m_2)$, $c_{iv}=k/(m_i(\gamma_i-1))$, and γ_i is the ratio of specific heats. For the interaction potential of solid spheres, we have the relation

$$K=\frac{16}{3}\frac{\rho_1\rho_2}{m_1m_2}\sqrt{\frac{k\pi}{2}}\sqrt{\frac{T_1}{m_1}+\frac{T_2}{m_2}}\,\delta_{12}^2,\qquad \delta_{12}=\frac{\delta_1+\delta_2}{2}$$

(δ_i is the diameter of a molecule of the ith gas).

For low (or zero) concentrations of the jth gas, we use the Euler equations for the pure ith gas and determine the parameters of the other gas from the relations $\dfrac{\partial n_j}{\partial t}+\dfrac{\partial n_ju_j}{\partial x}=0$, $u_j=u_i$, and $T_j=T_i$. The transition to the heavy gas is performed if the molar concentration of the light gas is $x_j=n_j/(n_i+n_j)<1\%$, and the reverse transition (to the light gas) occurs if the mass concentration of the heavy gas is $\alpha_j=\rho_j/(\rho_i+\rho_j)<1\%$.

Formation of the Mixing Region. Let us first describe the original mixing region. In the experiments of [4–6], such a region was formed in a shock tube by a rapidly removed plate separating the channel into two parts. As was shown in [5], the basic process governing mixing is molecular diffusion, which makes important obtaining an asymptotic solution for the initial mixing region on the basis of Eq. (1).

Let there be two quiescent gases with identical pressures in a channel divided by a partition at the initial time. When the partition is removed, diffusion mixing of gases occurs.

We assume that $K\gg1$, i.e., the relaxation time for velocities and temperature is small. We introduce the mean molar velocities and temperatures ($u=x_1u_1+x_2u_2$ and $T=x_1T_1+x_2T_2$) and the relative velocities and temperatures ($v=u_2-u_1$ and $\tau=T_2-T_1$). Let us pass to these variables in Eq. (1). It follows from (1) that v and T are of order $1/K$. Retaining quantities of the first and second order of smallness in the resultant equations, we obtain the following system:

$$\frac{\partial n}{\partial t}+\frac{\partial nu}{\partial x}=0,\qquad n\frac{\partial x_1}{\partial t}+nu\frac{\partial x_1}{\partial x}=\frac{\partial}{\partial x}\Big(Dn\Big(\frac{x_1x_2(m_2-m_1)}{m_1x_1+m_2x_2}\frac{1}{p}\frac{\partial p}{\partial x}+\frac{\partial x_1}{\partial x}\Big)\Big),$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\frac{\partial p}{\partial x}=0,\qquad \rho c_v\frac{\partial T}{\partial t}+\rho c_vu\frac{\partial T}{\partial x}+p\frac{\partial u}{\partial x}=0,$$

$$D=x_1x_2p/K,\qquad n=n_1+n_2,\qquad \rho=\rho_1+\rho_2,\qquad c_v=\alpha_1c_{1v}+\alpha_2c_{2v},\qquad p=knT.$$

Here, D is the binary diffusion coefficient.

This system has an exact solution

$$u=0,\qquad T=\text{const},\qquad n=\text{const},$$

$$x_1=\frac{1-\Phi(\eta)}{2},\qquad \Phi(\eta)=\frac{2}{\sqrt{\pi}}\int\limits_0^\eta \mathrm{e}^{-u^2}\,du,\qquad \eta=\frac{x}{2\sqrt{Dt}}, \tag{2}$$

which satisfies the initial distribution of molar concentration: $x_1=1$ for $x<0$ and $x_1=0$ for $x>0$. The solution obtained describes the process of diffusion mixing of two gases.

Following [10], we introduce the mixing-region width via the molar concentration as

$$L=2\int\limits_{-\infty}^{x_0(t)}\frac{x_2-x_2^0}{x_2(x_0(t))-x_2^0}\,dx+2\int\limits_{x_0(t)}^{+\infty}\frac{x_1-x_1^1}{x_1(x_0(t))-x_1^1}\,dx,$$

where x_i^0 and x_i^1 are the molar concentrations of the ith gas on different sides of the mixing region and $x_0(t)$ is the middle of the mixing region, i.e., the point where $x_1=x_2=(x_1^0+x_1^1)/2$. This definition of the mixing-region width

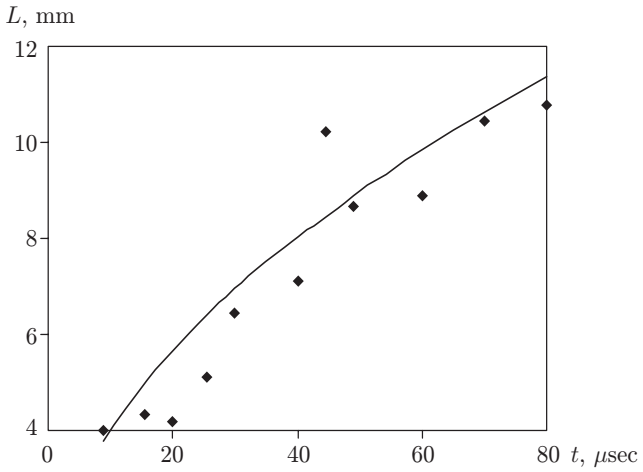

Fig. 1

is given here for the general case: for a mixture with different concentrations on both sides of the partition at the initial time.

To solve Eq. (2), we find the mixing-region width

$$L(t) = 4\Big(\int\limits_{-\infty}^{0} (1 - x_1)\, dx + \int\limits_{0}^{+\infty} x_1\, dx \Big). \tag{3}$$

Figure 1 shows the calculated (solid curve) and experimental [5] (points) dependences of the diffusion width of the mixing layer in a mixture of argon and helium versus time. It is seen that formulas (2) and (3) offer a satisfactory description of experimental data. The solution of this problem by the full model is in good agreement with the asymptotic solution (2), which allows one to calculate interaction of the layer of the mixture with the shock wave, using this asymptotic solution as the initial distribution of parameters in the layer.

Calculation Technique. As a method for spatial approximation of system (1), we use the method of flux-vector splitting [11]. To retain monotonicity of the solution in regions with high gradients, the order of approximation is reduced by a minmod limiter used for constructing TVD schemes [12]. Implicit approximation of the right sides of system (1) was used [9], which allowed us to avoid strengthening the restriction on the time step imposed by the Courant condition.

Interaction of the Diffusion Layer with the Shock Wave. Based on the considerations described above, we consider shock-wave transition through the layer formed owing to molecular diffusion. At the initial time, the concentration distribution is described by formulas (2). Actually, the mixing layer is located in the domain $-L_0/2 < x < L_0/2$, where L_0 is the initial width of the layer (gas No. 1 and gas No. 2 are located on the left and on the right, respectively). The shock wave is impinging onto the layer from the right; at the time $t = 0$, the shock wave is located at the point where the concentration of gas No. 1 is 1%. The parameters behind the shock-wave front are determined from the Rankine–Hugoniot relations for gas No. 2. The pressure ahead of the front is 0.5 atm.

Shock-Wave Transition from the Light Gas to the Heavy Gas. It is known that shock-wave transition from the light gas to the heavy gas through a contact discontinuity leads to formation of refracted and reflected shock waves. A similar picture is observed when the shock wave traverses the mixing layer. The presence of a transitional region, however, leads to certain specific features of the wave pattern of the flow.

Shock-wave transition from helium to argon is characterized by the following parameters: ratio of molecular weights $m_1/m_2 = 10$, $\delta_2/\delta_1 = 2.19/3.66$, and the Atwood number $\mathrm{A} = (\rho_h - \rho_l)/(\rho_h + \rho_l) = 0.82$ (ρ_h and ρ_l are the densities of the heavy and light gases at the initial time). Shock-wave transition from helium to xenon is characterized by the corresponding values $m_1/m_2 = 32.8$, $\mathrm{A} = 0.94$, and $\delta_2/\delta_1 = 2.19/4.94$. For the monatomic gases under consideration, we assumed that $\gamma_1 = \gamma_2 = 5/3$.

Figure 2 shows the relative width of the diffusion region L/L_0 versus time [the solid and dashed curves refer to the helium–argon and helium–xenon mixtures, respectively; curves 1 refer to $\mathrm{M} = 2.5$ and $L_0 = 13$ mm, curves 2

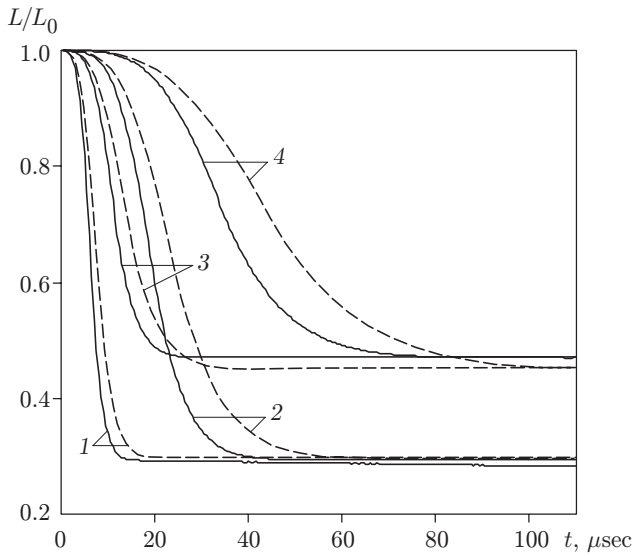

Fig. 2

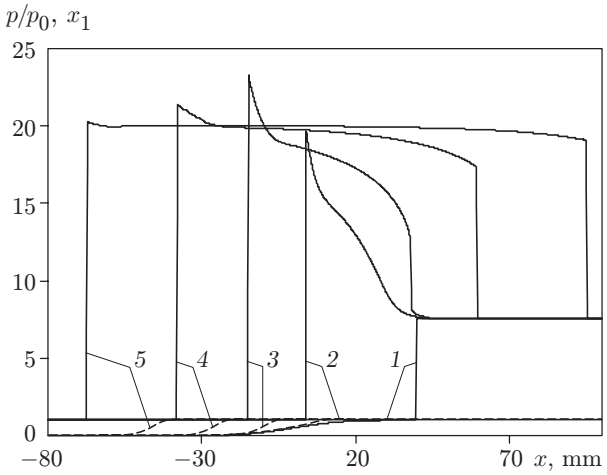

Fig. 3

refer to M = 2.5 and $L_0 = 40$ mm, curves 3 refer to M = 1.5 and $L_0 = 13$ mm, and curves 4 refer to M = 1.5 and $L_0 = 40$ mm. The layer compression is mainly determined by the Mach number and weakly depends on the ratio of molecular weights and initial width of the layer. As the Mach number changes from 1.5 to 2.5, the layer thickness decreases by a factor of 2.1 to 3.5. An increase in the initial width of the layer affects the compression time, which is caused by a greater time of shock-wave transition through the layer. The data presented are in agreement with the experimental results of [5], where a decrease in the layer thickness by a factor of 2 to 2.5 for a Mach number M = 2.5 was also noted.

The solid curves in Fig. 3 show the pressure distributions at different times for a shock wave passing from helium to xenon for M = 2.5 and $L_0 = 40$ mm: $t = 0$ (1), 20 (2), 40 (3), 70 (4), and 100 μsec (5).

As the shock wave traverses the layer, its intensity increases, and the pressure at the exit from the layer exceeds the value corresponding to the initially stepwise variation of density. The maximum difference in these values depending on the ratio of molecular weights is 12% for the helium–xenon mixture and 6% for the helium–argon mixture for M = 2.5. As the shock wave propagates over the light gas, the pressure decreases, and the pressure profile automatically transforms to that corresponding to the solution of the problem of shock-wave interaction with a contact discontinuity. A similar effect was noted in [5], where the experimental values of velocity of the

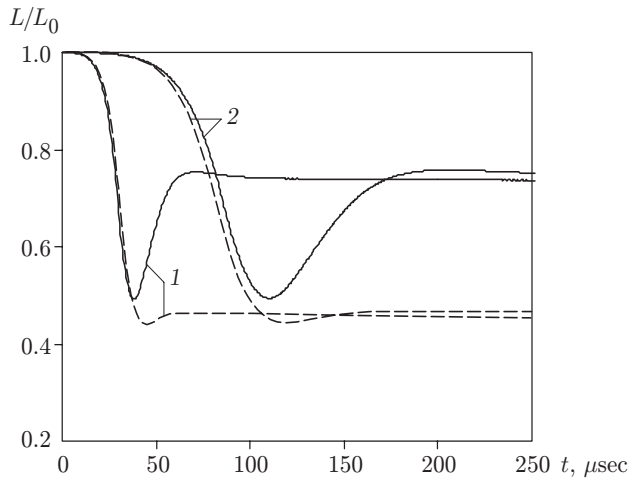

Fig. 4

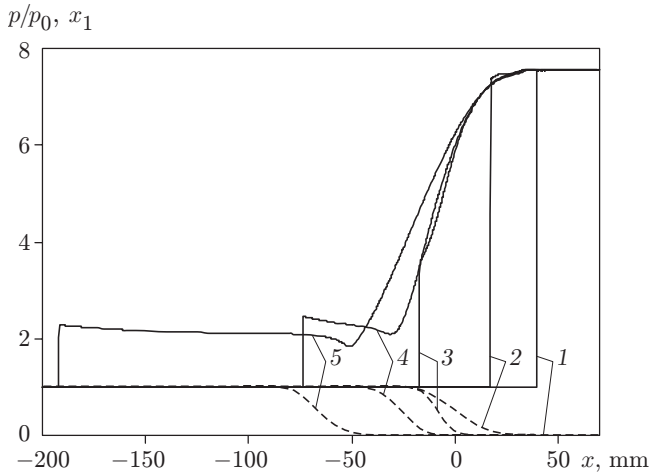

Fig. 5

refracted shock wave near the layer were 10% higher than the values calculated under the assumption that the density variation is discontinuous. As the shock wave traverses the mixing layer, a reflected compression wave is formed behind the shock wave; the reflected wave propagates into the light gas, and its collapse occurs outside the mixing layer. Figure 3 also shows the molar concentrations for helium (dashed curves) characterizing the positions of the mixing layer at different times.

Shock-Wave Transition from the Heavy Gas to the Light Gas. Shock-wave transition from the heavy gas to the light gas through the contact discontinuity is accompanied by formation of a refracted shock wave and an expansion wave.

We considered shock-wave transition from xenon to helium ($m_2/m_1 = 32.8$ and A = 0.94) and from xenon to argon ($m_2/m_1 = 3.28$ and A = 0.53). Figure 4 shows the relative width of the mixing region versus time for M = 2.5: the solid and dashed curves refer to the xenon–helium and xenon–argon mixtures, respectively; curves 1 and 2 refer to $L_0 = 13$ and 40 mm, respectively. The magnitude of compression depends on the ratio of molecular weights (Atwood number). For high values of Atwood number, the layer is strongly compressed after shock-wave transition, and its expansion occurs; as a result, the layer is compressed approximately by a factor of 1.5. As the ratio of molecular weights decreases, the effect of expansion after compression vanishes. The initial width of the layer affects only the compression time, as was observed in the case with shock-wave transition from the light gas to the heavy gas. Similar changes in the mixing-region thickness are also observed for other Mach numbers.

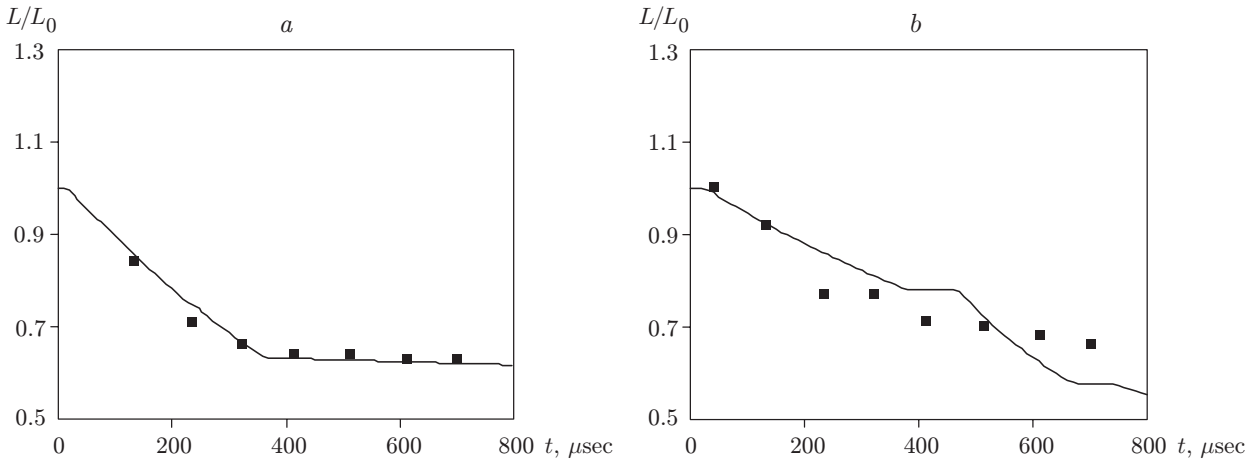

Fig. 6

Figure 5 shows the profiles of total pressure (solid curves) and molar concentration (dashed curves) of the light gas for M $= 2.5$ and $L_0 = 40$ mm at different times for shock-wave transition from helium to xenon: $t = 0$ (1), 50 (2), 110 (3), 150 (4), and 230 μsec (5). As the shock wave moves through the layer, its intensity decreases. At the exit from the layer ($t = 110$ μsec), however, the pressure behind the shock wave is significantly higher than that in the problem of decay of an arbitrary discontinuity. The mixing layer is overcompressed, and its expansion occurs, which results in formation of a compression wave propagating to the right and interacting with the expansion wave moving over the heavy gas. As the refracted wave moves over the light gas, the pressure behind this wave decreases and tends to the value corresponding to the solution of the problem of shock-wave interaction with the contact discontinuity between the heavy and light gases. Similar results were obtained for different Mach numbers and initial widths of the mixing region.

Interaction of the Layer with the Compression Wave. The Rayleigh–Taylor instability developed in the transitional layer was experimentally studied in [6]. A permanent acceleration was imparted to the contact region through the compression wave formed ahead of the flame front. In computations, the compression wave was simulated by a centered compression wave with parameters that imparted an acceleration of the order of $10^5 g$ (g is the acceleration of gravity) to the layer. The wave width was chosen such that no collapse occurred. Gas No. 2 in both computations and experiments was an oxygen–hydrogen mixture (molecular weight 18.5, $\gamma_2 = 1.4$, and $\delta_2/\delta_{\mathrm{He}} = 4.00/2.19$). Figure 6a shows the calculated (solid curve) and experimental (points) dependences of the relative width of the region of diffusion mixing versus time for compression-wave transition from the oxygen–hydrogen mixture into argon (from the light to the heavy gas). The computation results agree with experimental data. In this case, we see a compression wave passing into the heavy gas and a reflected compression wave emanating from the mixing layer and propagating to the right over the light gas.

In Fig. 6b, similar dependences are plotted for the case the compression wave passes from the oxygen–hydrogen mixture into helium (from the heavy to the light gas). In contrast to the experiment, the computations reveal a slower compression of the layer behind the transient shock wave and a stronger compression behind the shock wave reflected from the butt-end face. Apparently, these differences are caused by the fact that the heavy gas was located above the light gas in the vertical shock tube, i.e., the mixing layer was unstable, which excited two-dimensional disturbances ignored in computations. In this case, a compression wave propagating in the light gas and an expansion wave leaving the layer and moving over the heavy gas were observed. In contrast to a similar problem of shock-wave interaction with the mixing layer, the layer did not expand after transition of the incident compression wave through the layer.

Conclusions. A mathematical model of a two-velocity two-temperature mixture of gases is proposed for the description of processes inherent in interaction of shock and compression waves with the region of mixing of two gases. A solution is constructed within the framework of a simplified mathematical model, which describes the formation of a diffusion mixing layer. The problem of interaction of this layer with shock and compression waves in a one-dimensional unsteady flow is solved in the general case, for the full model of the mixture.

The wave patterns arising owing to shock-wave transition from the light to the heavy gas and from the heavy to the light gas are analyzed. It is found that the mixing layer is overcompressed if the shock wave passes from the heavy to the light gas, which leads to expansion of the mixing layer after compression behind the front of the refracted shock wave.

Satisfactory agreement of numerical data with the measured width of the mixing layer is obtained.

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